

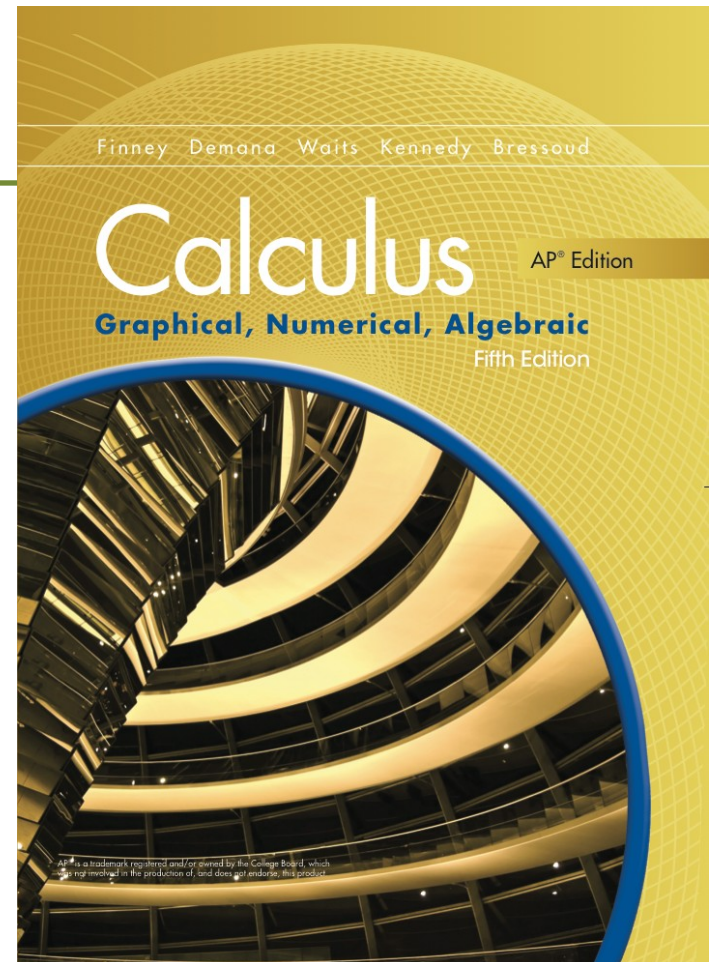
Chapter 1

Prerequisites for Calculus



Section 1.2

Functions and Graphs



Quick Review

In Exercises 1–6, solve for x .

1. $3x - 1 \leq 5x + 3$

2. $x(x - 2) > 0$

3. $|x - 3| \leq 4$

4. $|x - 2| \geq 5$

5. $x^2 < 16$

6. $9 - x^2 \geq 0$

Quick Review

In Exercises 7 and 8, describe how the graph of f can be transformed to the graph of g .

7. $f(x) = x^2$, $g(x) = (x+2)^2 - 3$

8. $f(x) = |x|$, $g(x) = |x-5| + 2$

Quick Review

In Exercises 9 – 12, find all real solutions to the equations.

9. $f(x) = x^2 - 5$

(a) $f(x) = 4$

(b) $f(x) = -6$

10. $f(x) = \frac{1}{x}$

(a) $f(x) = -5$

(b) $f(x) = 0$

Quick Review

11. $f(x) = \sqrt{x+7}$

(a) $f(x) = 4$

(b) $f(x) = 1$

12. $f(x) = \sqrt[3]{x-1}$

(a) $f(x) = -2$

(b) $f(x) = 3$

Quick Review

In Exercises 1–6, solve for x .

$$1. 3x - 1 \leq 5x + 3 \quad [-2, \infty)$$

$$2. x(x - 2) > 0 \quad (-\infty, 0) \cup (2, \infty)$$

$$3. |x - 3| \leq 4 \quad [-1, 7]$$

$$4. |x - 2| \geq 5 \quad (-\infty, -3] \cup [7, \infty)$$

$$5. x^2 < 16 \quad (-4, 4)$$

$$6. 9 - x^2 \geq 0 \quad [-3, 3]$$

Quick Review

In Exercises 7 and 8, describe how the graph of f can be transformed to the graph of g .

7. $f(x) = x^2$, $g(x) = (x + 2)^2 - 3$

2 units left and 3 units downward

8. $f(x) = |x|$, $g(x) = |x - 5| + 2$

5 units right and 2 units upward

Quick Review

In Exercises 9-12, find all real solutions to the equations.

9. $f(x) = x^2 - 5$

(a) $f(x) = 4$

(a) $-3, 3$

(b) $f(x) = -6$

(b) no real solution

10. $f(x) = \frac{1}{x}$

(a) $f(x) = -5$

(a) $-\frac{1}{5}$

(b) $f(x) = 0$

(b) no real solution

Quick Review

11. $f(x) = \sqrt{x+7}$

(a) $f(x) = 4$

(a) 9

(b) $f(x) = 1$

(b) - 6

12. $f(x) = \sqrt[3]{x-1}$

(a) $f(x) = -2$

(a) - 7

(b) $f(x) = 3$

(b) 28

What you'll learn about

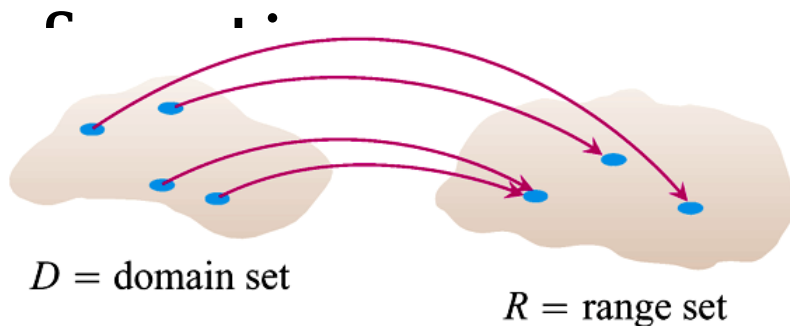
- Function, domain, and range
- Interval notation
- Graphs in the coordinate plane
- Odd and even symmetry
- Piece-wise defined functions, including absolute value
- Composite Functions

...and why

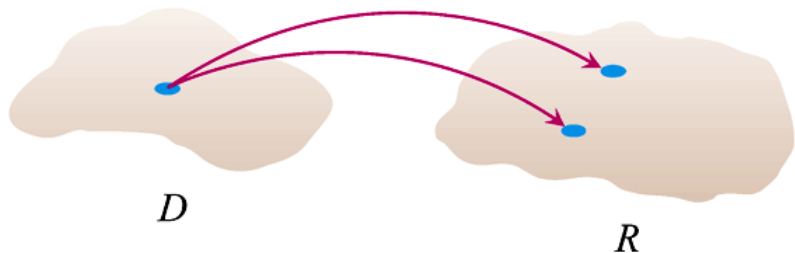
Functions and graphs form the basis for understanding mathematics applications.

Function and Related Terms

A **function** from a set D to a set R is a rule that assigns a unique element in R to each element in D . The set of all input values in D is the **domain** of the function, and the set of all output values in R is the **range** of the



(a)



(b)

Function and Related Terms

In **function notation**, we use $f(x)$ to denote the range value that f assigns to the domain value x . The set of all points (x, y) in the coordinate plane determined by the rule $y = f(x)$ is the **graph of the function f** . The variable x is the **independent variable** and the variable y is the **dependent variable**.

Domains and Ranges

When we define a function $y = f(x)$ with a formula and the domain is not stated or restricted by context, then the **implied domain** is the largest set of x values for which the formula gives real y values.

The restricted domain is called the **relevant domain**.

Domains and Ranges

The domains and ranges of many real-valued functions of a real variable are intervals or combinations of intervals.

The intervals may be open, closed, or half-open and finite or infinite.

Intervals

The endpoints of an interval make up the interval's **boundary** and are called **boundary points**. The remaining points make up the interval's **interior** and are called **interior points**. **Closed intervals** contain their boundary points. **Open intervals** contain no boundary points. Every point of an open interval is an interior point of the interval.

Intervals



Name: Open interval ab

Notation: $a < x < b$ or (a, b)



Name: Closed interval ab

Notation: $a \leq x \leq b$ or $[a, b]$

Intervals



Closed at a and open at b

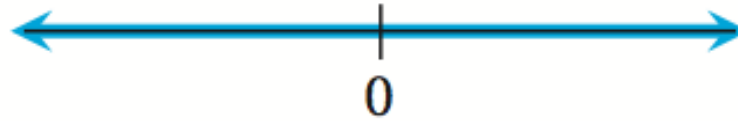
Notation: $a \leq x < b$ or $[a, b)$



Open at a and closed at b

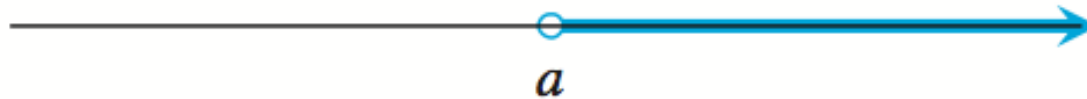
Notation: $a < x \leq b$ or $(a, b]$

Intervals



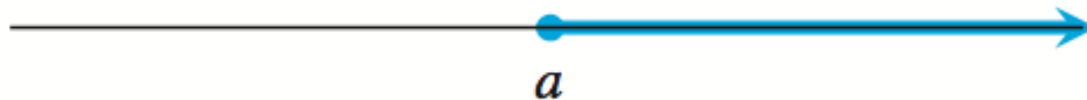
Name: The set of all real numbers

Notation: $-\infty < x < \infty$ or $(-\infty, \infty)$



Name: The set of numbers greater than a

Notation: $a < x$ or (a, ∞)



Name: The set of numbers greater than
or equal to a

Notation: $a \leq x$ or $[a, \infty)$

Intervals



Name: The set of numbers less than b

Notation: $x < b$ or $(-\infty, b)$



Name: The set of numbers less than
or equal to b

Notation: $x \leq b$ or $(-\infty, b]$

Graph Viewing Skills

1. Recognize that the graph is reasonable.
2. See all the important characteristics of the graph.
3. Interpret those characteristics.
4. Recognize grapher failure.

Example

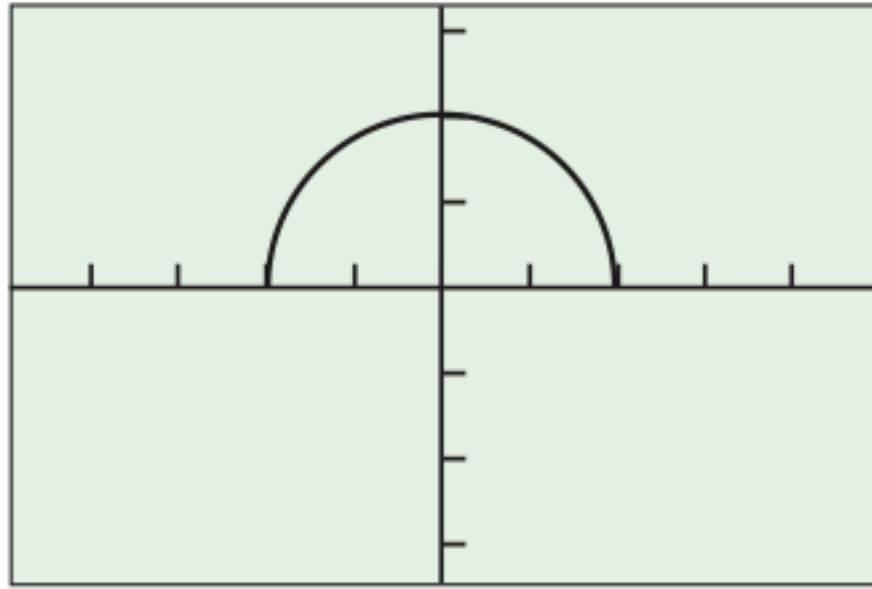
Use a grapher to identify the domain and range, and then draw a graph of the function.

(a) $y = \sqrt{4 - x^2}$

(b) $y = x^{2/3}$

Example - Solution

(a) $y = \sqrt{4 - x^2}$



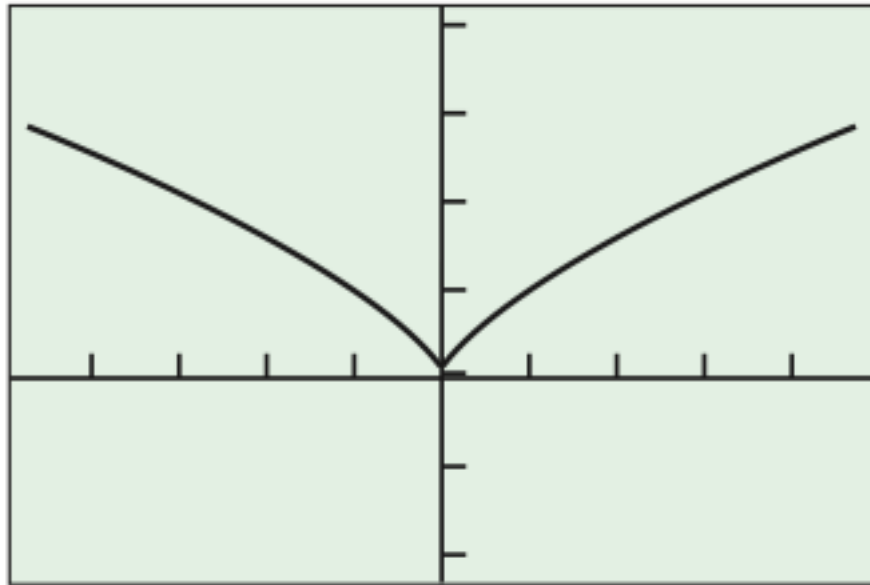
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Domain: $[-2, 2]$

Range: $[0, 2]$

Example - Solution

(b) $y = x^{2/3}$



$[-4.7, 4.7]$ by $[-2.1, 4.1]$

Domain $(-\infty, \infty)$

Range $[0, \infty)$

Even Functions, Odd Functions

A function $y = f(x)$ is an

even function if $f(-x) = f(x),$

odd function if $f(-x) = -f(x),$

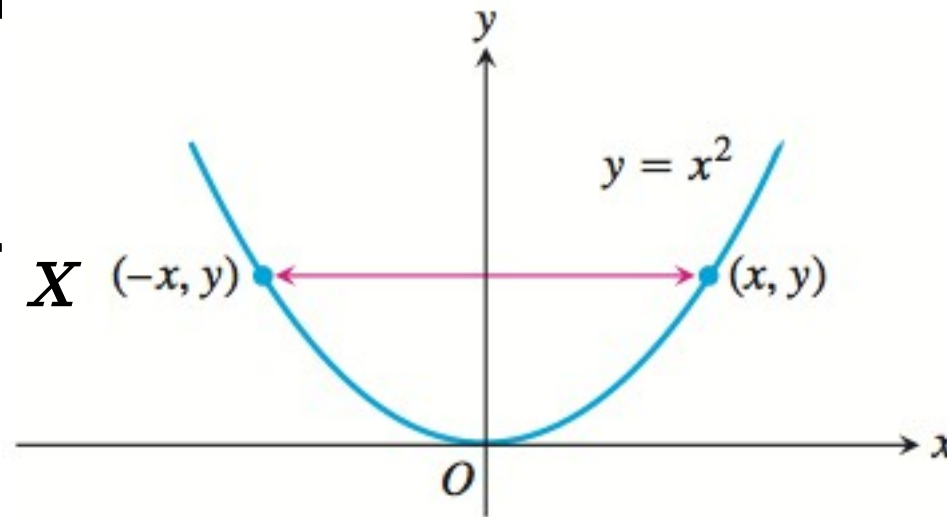
for every x in the function's domain.

Even Functions

The graph of an even function is **symmetric about the x -axis**.

Since $f(-x) = f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, y)$ lies on the graph.

Note the power of x is even.

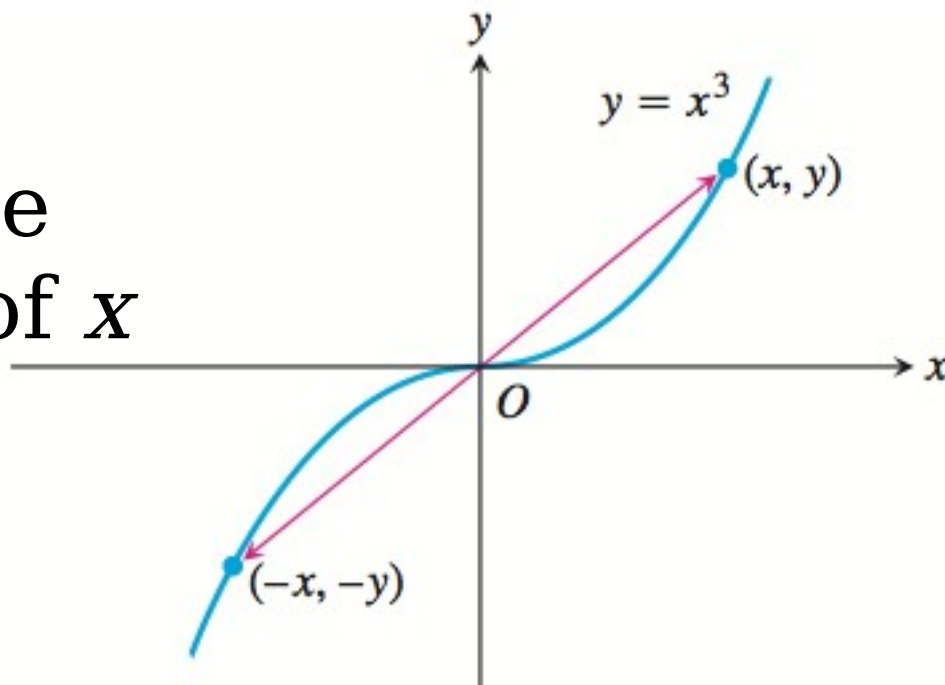


Odd Functions

The graph of an odd function is **symmetric about the origin**.

Since $f(-x) = -f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, -y)$ lies on the graph.

Note the power of x is odd.



Piecewise-Defined Functions Example

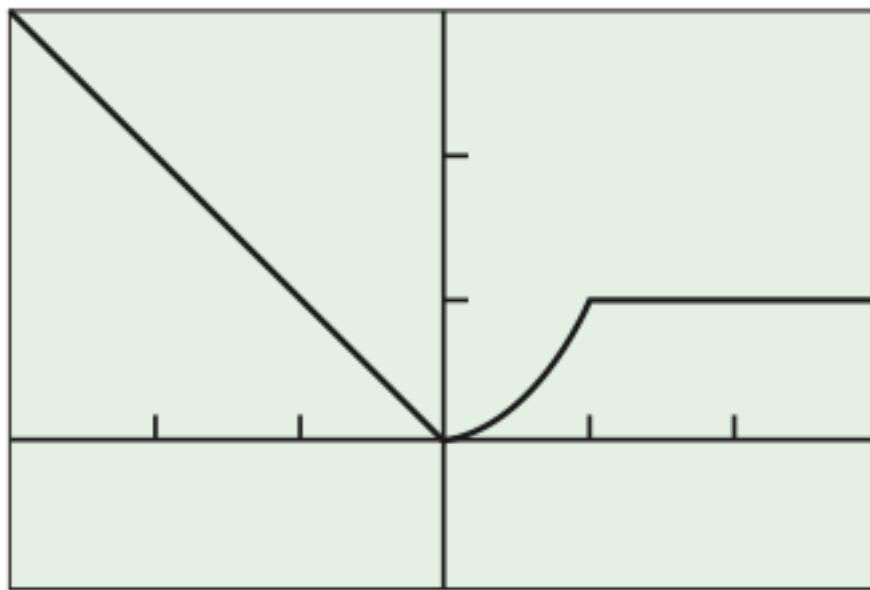
Graph

$$f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1. \end{cases}$$

Example - Solution

Graph

$$f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1. \end{cases}$$

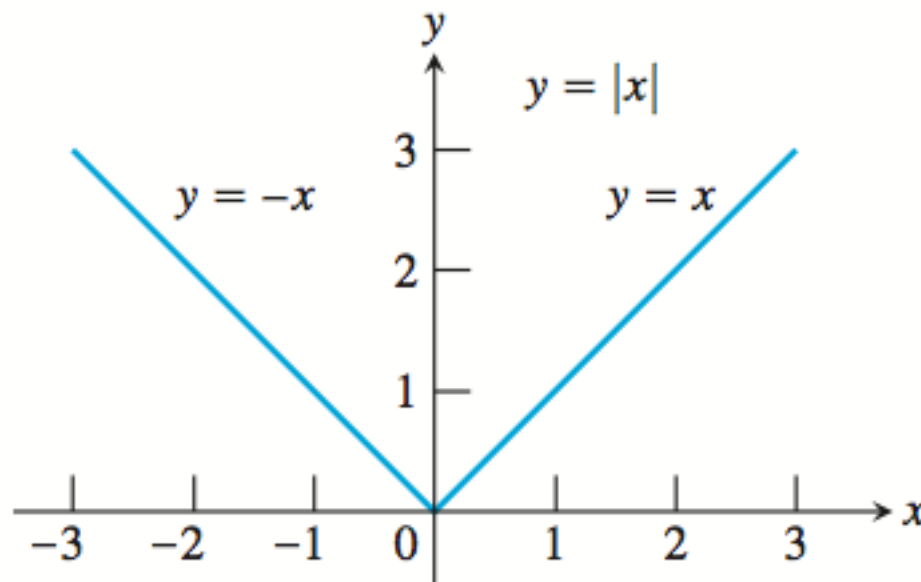


$[-3, 3]$ by $[-1, 3]$

Absolute Value Function

The **absolute value function** $y = |x|$ is defined piecewise by the formula

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0. \end{cases}$$



Composite Functions

We say that the function $(f \circ g)(x)$ (read ***f of g***) is the **composite of g and f** .

It is made by *composing* g and f in the order of g first, then f .

The notation for this composite is $(f \circ g)$, read as “ f of g .” Thus, the value of $(f \circ g)$ at x is $(f \circ g)(x) = f(g(x))$.

Example

Find a formula for $f(g(x))$ if $f(x) = x - 7$ and $g(x) = x^2$. Then find $f(g(2))$.

Example - Solution

Find a formula for $f(g(x))$ if $g(x) = x^2$ and $f(x) = x - 7$. Then find $f(g(2))$.

$$\begin{aligned} f(g(x)) &= g(x) - 7 \\ &= x^2 - 7 \end{aligned}$$

$$f(g(2)) = (2)^2 - 7 = -3$$